Maths Outside HTQ Worksheet – Dimensional Analysis

Notes:

- This worksheet involves several questions for dimensional analysis. The questions are written out on the following page
- In terms of repeating variables use the ones you know are common first then look at the others
- I suggest printing this sheet to make it easier to work on

Question 1

$$K_c \propto {D_{AB}}^a \rho^b v^c \mu^d L^e$$

K_c	Mass Transfer Coefficient $[LT^{-1}]$
D_{AB}	Mass diffusivity $[L^2T^{-1}]$
ρ	Fluid density $[ML^{-3}]$
v	Fluid velocity $[LT^{-1}]$
μ	Dynamic viscosity$[ML^{-1}T^{-1}]$
L	Characteristic length $[L]$

a. Work out the dimensionless groups

b. A laboratory experiment is carried out using a small test surface in flowing water. The laboratory system has:

Variable	Lab value
Characteristic length	0.050
Fluid velocity	0.02
Fluid density	1000
Dynamic viscosity	$1.0 * 10^{-3}$
Mass diffusivity	$1.5 * 10^{-9}$
Measured mass-transfer coefficient	$2.0 * 10^{-5}$

The real industrial system will have a characteristic length of: $L_{Real} = 0.5$ and will use the **same fluid at the same temperature**.

For each of your dimensionless groups calculate their value:

π_1:
π_2:
π_3:

c. For the real system to behave similarly to the lab system the 3 dimensionless groups must stay the same. Based on this information work out the fluid velocity of the new system (the same fluid is being used so keep the Fluid density & Dynamic viscosity the same)

d. Using the remaining dimensionless group, calculate the expected mass-transfer coefficient of the real system.

Question 2

$$h \propto k^a D^b \rho^c v^d \mu^e {c_p}^f$$

h	Convective heat-transfer coefficient $[MT^{-3}\theta^{-1}]$
k	Thermal conductivity $[MLT^{-3}\theta^{-1}]$
D	Characteristic diameter $[L]$
ρ	Fluid density $[ML^{-3}]$
v	Fluid velocity $[LT^{-1}]$
μ	Dynamic viscosity $[ML^{-1}T^{-1}]$
c_p	Specific heat capacity $[L^2T^{-2}\theta^{-1}]$

a. Use Buckingham π theorem to determine the dimensionless groups.

A small electronic cooling system is tested in a laboratory.

Variable	Lab value
$\boldsymbol{D}$	0.020
$\boldsymbol{v}$	2.0
$\boldsymbol{\rho}$	1.2
$\boldsymbol{\mu}$	$1.8 * 10^{-5}$
$\boldsymbol{k}$	0.026
$\boldsymbol{c_p}$	1005
$\boldsymbol{h}$	65

b. For each of your dimensionless groups calculate their value:

π_1:
π_2:
π_3:

c. The real cooling system has: $\boldsymbol{D_{real} = 0.100m}$ and uses the same air at the same temperature. Calculate the air velocity required in the real system to maintain similarity with the laboratory system.

d. Calculate the expected real-life heat-transfer coefficient h.

Question 3

A company is designing a new water distribution system for an industrial site. Before installing the full-size pipework, the engineers build a smaller laboratory model so they can investigate how much pressure will be lost as water flows through the system.

The engineers believe that the pressure drop through a straight section of pipe is affected by the density and viscosity of the water, the average velocity of the flow, the diameter of the pipe, and the length of the pipe section.

The laboratory model uses a pipe with a diameter of 0.020m and a length of 0.400m. Water flows through it at 0.50m/s. The water has a density of $1000 kg/m^{-3}$ and a dynamic viscosity of $1.00 * 10^{-3}$Pas. During testing, the engineers measure a pressure drop of 2500Pa.

The real installation will use pipe with a diameter of 0.100m. The engineers want the full-size system to behave in a dynamically similar way to the laboratory model.

a. Using **dimensional analysis and Buckingham π theorem**, determine the dimensionless groups that describe the system.

b. You should then use these groups to determine:
   - how long the equivalent section of full-size pipe should be;
   - the velocity the water should travel at in the full-size system;
   - the pressure drop you would expect across that section of pipe.

   Assume that the full-size system uses the **same water at the same temperature** as the laboratory test.

   You should clearly show how maintaining the same dimensionless groups allows the laboratory model to be used to predict the behaviour of the real installation.